Solar Energy Integration Lesson

Introduction

Solar energy production has seen fast gains in the past few years. The falling cost of photovoltaic (PV) cells and increased political support are major drivers of these gains.

Some of the advantages of solar power are the following:
1. Cleaner production of energy compared to conventional sources such as natural gas and coal.
2. Near zero operating costs. Light from the sun is free and maintenance costs are low.

However, despite the attractiveness of solar energy in light of the current environmental and climate crisis, solar energy still has challenges to overcome. Some of these challenges are:
1. Solar panels do not produce electricity at night; People use electricity at night. Since the electric grid currently does not have the energy storage capabilities needed to store some of the energy produced during the day and use it to supply energy during the night, we need to find ways to do so.
2. Investment costs are still relatively high.
3. Its production is, to some extent, hard to predict. Although we are pretty good at predicting sunrise and sunset, predicting when a cloud is going to cover the sky is hard.

Purpose

The purpose of this lesson is to introduce the students to the challenges of integrating solar energy to the electricity grid with the use of numerical simulations. More specifically, it is meant to shed light on the advantages (mentioned in the introduction of this document) and on the challenges (challenges 1 and 2 from the introduction) of solar energy.

Objectives

The students will be familiar with the following concepts:
- Electric load, load profile, and net load
- Fixed and flexible load
- Solar power
- Electricity production costs
- Optimization

The skills that the students will learn/strengthen are:
- Interpreting and extracting information from plots and graphs
- Adjust parameters and run numerical simulations

- Solve word problems using a numerical simulation package
- Translate policy into numerical simulations
- Recognize a small optimization problem

Concepts/ vocabulary

Electrical Grid

An electrical grid is an interconnected network for delivering electricity from suppliers to consumers. It consists of generating stations that produce electrical power, high-voltage transmission lines that carry power from distant sources to demand centers, and distribution lines that connect individual customers.

https://en.wikipedia.org/wiki/Electrical_grid

Electrical load

An electrical load, or simply load, is an electrical component or portion of a circuit that consumes electric power. In electric power circuits examples of loads are appliances and lights.
https://en.wikipedia.org/wiki/Electrical_load
https://en.wikipedia.org/wiki/Electric_power_system

Load profile

A load profile is the variation in the electrical load versus time. A load profile will vary according to customer type (typical examples include residential, commercial and industrial), temperature and holiday seasons.
https://en.wikipedia.org/wiki/Load_profile

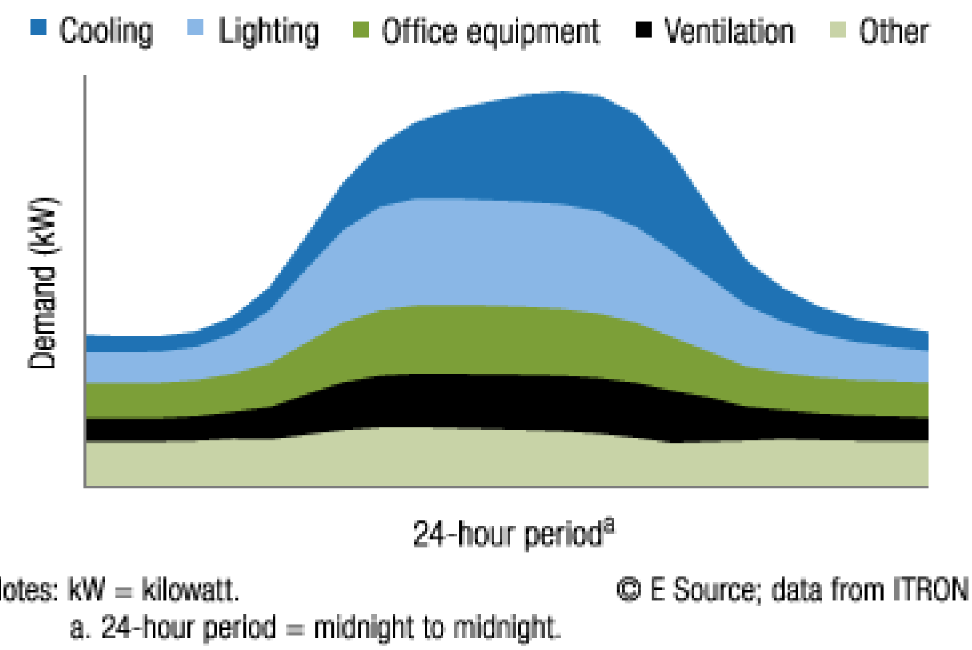


Figure 1: Load profile for a typical California office building. Source: https://bizenergyadvisor.com/large-offices.

Fixed load and flexible load

Fixed load is the amount of load that *has* to be served at each particular hour. Flexible load is load that does not have to be consumed at a particular time of day.

For instance hospital equipment is critical to hospital patients and thus *has* to run when needed. This is a fixed load. Or, a little bit less extreme case, a T.V. that *has* to be on at 9 PM every Sunday to watch Game of Thrones is also considered a fixed load.

Flexible loads on the other hand can be shifted around during the day. For instance, a factory may choose to run either a day shift or a night shift depending on which one is more convenient. Another example would be a dish washer. One may run it in the morning or in the evening, again, depending on which one is more convenient.

Solar power

Solar power is the conversion of sunlight into electricity, either directly using photovoltaics, or indirectly using concentrated solar power.
https://en.wikipedia.org/wiki/Solar_power

Solar power profile

A solar power profile is the variation in the solar power production versus time. Typically, the solar power production starts when the sun comes out in the morning, peaks at noon, and goes to zero at sunset. Clouds, shades, and seasons affect solar power production.

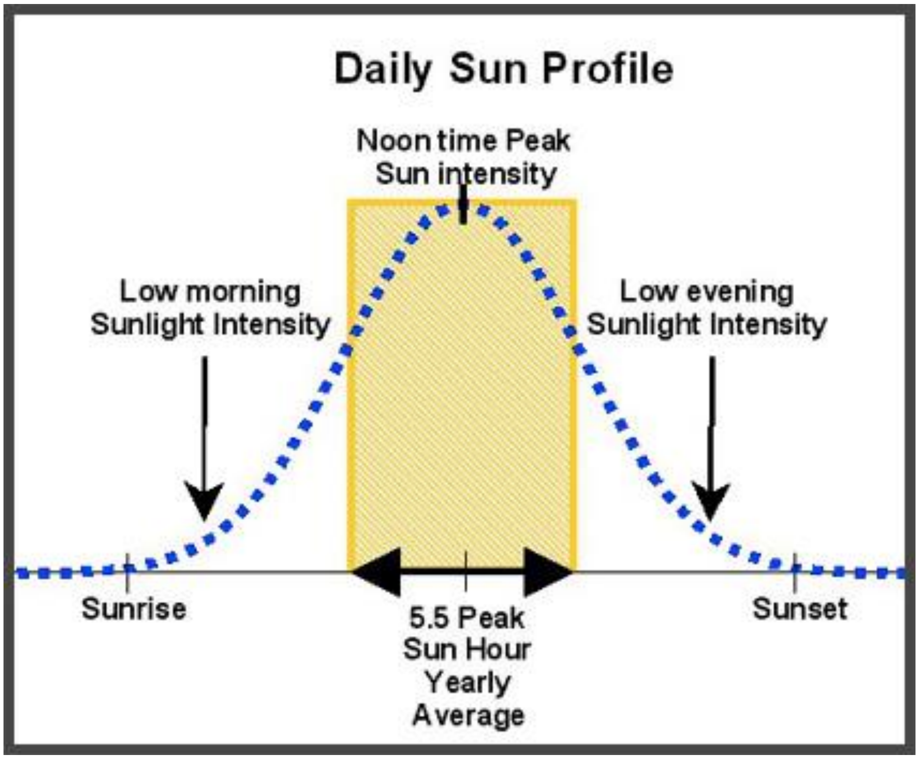


Figure 2: Typical solar power profile. Source: http://www.seslb.com/index.php?dbs=productsview&v=209

Net load

In this lesson, net load is the difference between the load and solar power. It is the portion of the load that has to be served by other power sources.

Production cost

Throughout this lesson, the production cost of electricity is the variable cost of serving the load during the day.

Optimization

Optimization is the selection of the best element from a set of possibilities.
Example:
You are at Gasworks Park on a sunny day. You have the option of sitting under a tree or sitting under the sun. You select sitting under the sun since this is a relatively rare occurrence in Seattle and you would like to get some sun. You just optimized your sitting position from two possible options.

In a more mathematical sense, when one performs optimization, one seeks to maximize or minimize a mathematical function – the objective function – while satisfying a set of equations – the constraints.

https://en.wikipedia.org/wiki/Mathematical_optimization

Mathematical notation

Units

Watts (W): unit of power equal to one joule per second.
Watt-hour (Wh): unit of electric energy. Equivalent to consuming one watt during one hour.

Fixed load

The fixed load, denoted by $load^{Fixed}(t)$, is the amount of energy that has to be consumed at time t.

Example:
A T.V. (100 kW) must run at both $t = 1$ and $t = 2$ but a washing machine can consume its required 500 Wh at any of those two times, then: $load^{Fixed}(1) = 100$ Wh and $load^{Fixed}(2) = 100$ Wh.

Flexible load

The flexible load, denoted by $load^{Flex}$, is the amount of the total load can be consumed at any hour of day. The flexible load can be expressed as
$load^{Flex} = \alpha \cdot load^{total}$ where α is the proportion of flexible load and $load^{total}$ is the total load to be consumed throughout the day.

Example:
The flexible load in our previous example is the washing machine, which has to consume 500 Wh at any time. Then $load^{Flex} = 500$ Wh. The total load is $load^{total} = 100 + 100 + 500$
Wh. The proportion of flexible load is $\alpha = \frac{500}{100+100+500} = \frac{5}{7}$.

Solar generation

The solar generation, denoted $solar(t)$, is the amount of solar energy produced at hour t.

Example:
Suppose we have a solar panel with maximum capacity of 1 kW. If the sun comes out at 6 am, $solar(t)$ is zero during hours 1,2,3,4,5. From 6 am to 7 am, $solar(6)$ is 100 Wh. From noon to 1 pm, during a sunny day, it produces at capacity so $solar(12) = 1$ kWh.

Net load

The net load at time t is $load^{net}(t) = load(t) - solar(t)$. Where $load(t)$ is the amount of flexible and fixed load consumed at time t.

Example:

Suppose that the load of a house from 7 am to 8 am consists of an A.C. unit (1 kWh), and a coffee pot (0.1 kWh). Suppose that this house has a solar panel that from 7 AM to 8 AM produces 0.5 kWh. The net load from 7 to 8 AM is $load^{net}(t) = 1 + \ 0.1 - 0.5 = 0.6$ kW.

Production cost

The electricity production cost for each hour is given by: $cost(t) = a + b \cdot load^{net}(t) + c \cdot load^{net}(t)^2$. Where a, b, and c are cost parameters (all strictly positive).
The total production cost during one day is $cost^{total} = \sum_{t=1}^{24} cost(t)$.
Note that since only the cost of serving the net load is taken into account, it is implicitly assumed that the variable cost of solar energy is zero.

Example:
Suppose that the production cost parameters are $a = b = c = 1$.
Suppose that the load consists of a fridge (1 kW) that is turned on from 7 to 9 am and an A.C. (1kW) that is turned on from 8 – 9 am. The load from 7 to 8 is 1 kWh and the production cost for this hour is $cost(7) = 1 + 1 \cdot 1 + 1 \cdot 1^2 = \$3$. The load from 8 to 9 is 2 kWh and the production cost for this hour is $cost(7) = 1 + 1 \cdot 2 + 1 \cdot 2^2 = \$7$. The total production cost for both hours is $\$3 + \$7 = \$10$.

Cost minimization problem

The purpose of this optimization problem is to allocate the flexible load such that the total production cost is minimized.

The following is a quadratic program and can be solved with commercial software packages such as: excel, MATLAB, CPLEX, Gurobi, etc.

$$\text{minimize } \{cost^{total}\} \qquad (1)$$

Subject to:

$$cost^{total} = \sum_{t=1}^{24} \{a + b \cdot load^{net}(t) + c \cdot load^{net}(t)^2\} \qquad (2)$$

$$load^{net}(t) = load^{Fixed}(t) + \ x(t) - solar(t) \ \ \forall\, t = 1, \dots, 24 \qquad (3)$$

$$\sum_{t=1}^{24} x(t) = \alpha \cdot load^{total} \qquad (4)$$

$$x(t) \geq 0 \ \forall\, t = 1, \dots, 24 \qquad (5)$$

Here $x(t)$ are the decision variables and represent the amount of flexible load consumed at time t.

Equation (1) is the objective to be minimized (cost). Equation (2) defines the total cost of production. Equations (3) defines the net load as the difference between load consumed at each hour and the solar production at each hour. Equation (4) requires all the required flexible load to be consumed. Equation (5) are technical constraints that require the flexible load to be greater than or equals to zero at all times – this is equivalent to saying that flexible load can not produce energy.

Example:
Consider that we are trying to allocate the flexible load in two hours such that our production cost is minimized. Assume $a = b = c = 1$. Our fixed load is a laptop (0.1 kW) during the first hour and a T.V. (0.3 kW) during the second hour. Our flexible load is a coffee pot that has to consume 0.1 kWh at any time. No solar power is generated.

By inspection, we see that since the cost is quadratic, the coffee pot should be on during the first hour. Giving us a net load of 0.2 kWh during the first hour and 0.3 kWh during the second hour

$cost(1) = 1 + 0.2 + 0.2^2 = \$1.24$

$cost(2) = 1 + 0.3 + 0.3^2 = \$1.39$

$cost^{total} = cost(1) + cost(2) = \$2.63$

More formally, we can express the problem as

$$\text{minimize} \sum_{t=1}^{2} \{1 + load^{net}(t) + load^{net}(t)^2\}$$

Subject to:

$load^{net}(1) = 0.1 + x(1)$

$load^{net}(2) = 0.3 + x(2)$

$x(1) + x(2) = 0.1$

$x(1) \geq 0$

$x(2) \geq 0$

Substituting $x(2) = -x(1) + 0.1$ in the period 2 net load equation.

$$load^{net}(2) = 0.4 - x(1)$$

Substituting $x(2) = -x(1) + 0.1$ in the $x(2)$ inequality constraint $x(1) \leq 0.1$

Substituting the net loads in the objective function, we get:

$$\text{minimize } \{1 + .1 + x(1) + \left(.1 + x(1)\right)^2 + 1 + .4 - x(1) + \left(.4 - x(1)\right)^2\}$$

which is still subject to the constraints $x(1) \geq 0$ and $x(1) \leq .1$.

From calculus, we know that in order to minimize we need to set its derivative to zero. The derivative of the objective function is:

$1 + .1 + 2 \cdot x(1) - 1 - .8 + 2 \cdot x(1) = -.7 + 4 \cdot x(1) = 0$

Solving for $x(1)$ we get $x(1) = 0.175$. We know that the objective function is minimized since its second derivative is positive.

However, the constraint $x(1) \leq 0.1$ is violated by this solution. We thus settle with the closest feasible point, which is $x(1) = 0.1$ and $x(2) = 0$.

What this result means is that the flexible load should be consumed in the first period. In other words, we should make coffee during the first period and drink it during the second.

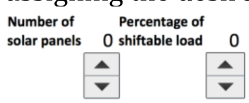
For a rigorous proof that this is in fact the optimal point, one needs to check that the Karush-Kuhn-Tucker conditions (https://en.wikipedia.org/wiki/Karush%E2%80%93Kuhn%E2%80%93Tucker_conditions) are in fact met. Showing this is beyond the scope of this lesson.

Note that even this very simple two period example can be fairly tedious to solve by hand. For our 24 hour example, an optimization solver is employed.

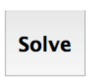
Numerical simulation tutorial

The cost minimization problem described previously is implemented in excel. The following is a tutorial of the numerical simulation.

Instructions

Simulate the system by assigning the desired characteristics (number of solar panels and amount of flexible load 

) to the system and minimizing the total system cost.

Press 

every time you change the system characteristics. IMPORTANT: MAKE SURE TO WAIT UNTIL THE SOLVER IS DONE BEFORE MAKING ANY OTHER CHANGES. The solver should run for a few seconds.

Plots

This excel spread-sheet shows two plots. The left one shows the non-optimized load, non-optimized net load, and solar power. The one on the right shows the optimized load, optimized net load, and solar power. Keep reading for further details.

Number of solar panels

The number of solar panels can be varied from 0 to 10. The solar production varies linearly with the number of solar panels. Each solar panel has a peak power of approximately .5 kW.

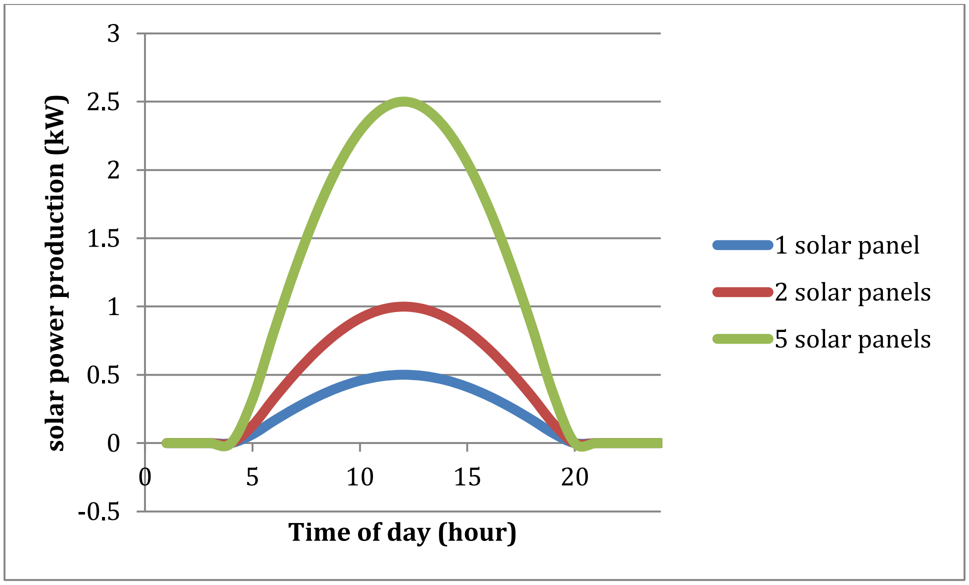


Figure 3: Solar power production for various numbers of solar panels

Percentage of flexible load
The percentage of flexible load can be varied from 0% to 100% in 5% increments. The value of α is related to the percentage of flexible load as follows:

$$\alpha = \frac{percentage\ of\ flexible\ load}{100}$$

Not-optimized load and net load

The not-optimized load given by the parameter $load(t) = load^{fixed}(t) + x(t)$.

Here, the flexible load $x(t)$ is distributed proportionally across the day. No cost minimization is performed to allocate the flexible load.

The not-optimized net load given by:
$load^{net}(t) = load^{fixed}(t) + x(t) - solar(t)$.

Optimized load and net load.

The optimized load is given by
$load(t)^* = \alpha \cdot load(t) + \ x(t)^*$ where $load(t)^*$ and $x(t)^*$ are obtained by solving the cost minimization problem.

The optimized net load is given by
$load^{net}(t)^* = \alpha \cdot load(t) + \ x(t)^* - solar(t)$.

Spilled solar energy
The system needs to "spill" or waste solar energy when the solar energy production is greater than the load.

The amount of spilled solar energy (not optimized) is given by $\sum_{t=1}^{24} \max(0, solar(t) - load(t))$.

The amount of spilled solar energy (optimized) is given by $\sum_{t=1}^{24} \max(0, solar(t) - load(t)^*)$.

Worksheet

Question 1
a)
Suppose that our electric system has no solar power and no flexible load. Answer the following questions:
a.1) What is the production cost for the non-optimized system? And for the optimized? Why?

<u>Answer: $257.02 for both. Both are equal because no flexible load can be moved around by the optimization problem to reduce the cost of serving the load.</u>

a.2) Looking at the plots, roughly, what is the difference between the maximum (peak) load and the minimum load (in kW units)?

<u>Answer: roughly 2 kW.</u>

b)
Increase the number of solar panels to three. Answer the following questions:

b.1) What is the peak solar power production (roughly)?
<u>Answer: 1.5 kW</u>
b.2) What is the difference between peak net load and minimum net load (not-optimized)?
<u>Answer 2.7 kW</u>
b.3) Suppose that your system is constrained by its ability to ramp up non-solar production (i.e. increase non-solar production from one hour to the next). Engineers have determined that for every 1 kW of minimum and maximum load difference, a new gas turbine has to be installed in order to ramp up non-solar production. How many gas turbines does your system need in case if the system has no solar panels? How many does it need if the system has 3 solar panels?

<u>Answer: 2 turbines if the system has no solar panels. 3 turbines if the system has 3 solar panels.</u>

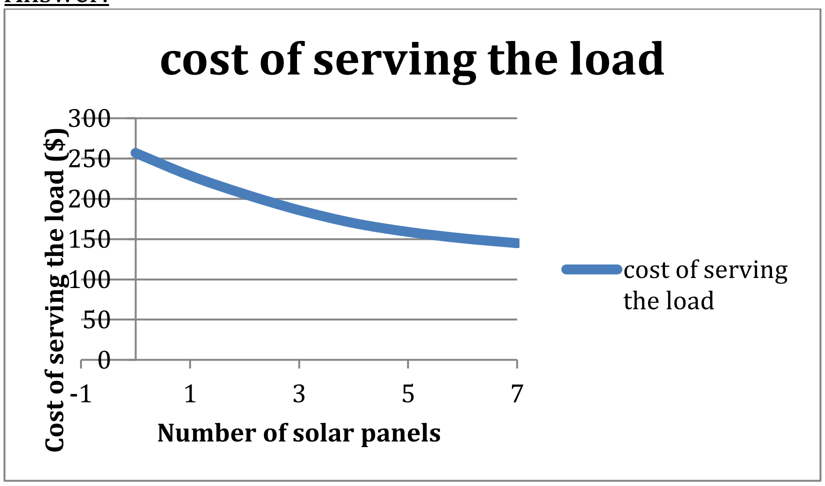
Question 2.
a)
a.1) If the system has no flexible load, how many solar panels can be installed without having to spill solar energy?
<u>Answer: 4 solar panels.</u>

a.2) Ignore the fixed costs of solar panels. Provide the cost of a system with no solar power and the cost of a system with 4 solar panels (no flexible load in both cases).

Answer: No solar panels cost is $257.02. System with 4 solar panels cost $170.45.

a.3) Suppose that newly elected president Bernie Sanders mandates that the system should have 6 solar panels. What percentage of flexible load is needed such that no solar power is spilled?
(Hint: the Spilled solar energy (optimized) must be zero).
Answer: 10%.

Question 3
a.1) Suppose that our system does not have flexible load. Plot the cost of serving the load as a function of number of solar panels for n=0,1,2,3,4,5,6,7.
Answer:



a.2) Engineers have calculated that the daily fixed cost of a solar panel is $10. If the total cost of the system is the cost of serving the load + the fixed costs, how many solar panels should you install in order to make the system most economical? (Assume that the system has no flexible load and that spilling solar energy is not an issue).
Answer: 5 solar panels

a.2) Engineers have decided on installing 5 solar panels. They have been contacted by a company that assures them that the equipment required to make 1% of the load flexible costs only $1 per day. In 5% increments, how much flexible load should the system install?

Answer: 15%.

Question 4

a.1) After building a wall at the southern border (Mexico paid) and banning all Muslims from entering the U.S., newly elected Dear Leader Donald Trump decided that he would deliver on his campaign promise of revitalizing the coal industry. To do so, he has provided subsidies to the coal industry that equal 20% of the cost of serving the load. Additionally, scientists estimate that the environmental costs of running this coal-based system are $100 per day. Calculate the total societal cost of running this system (Hint: the sum of cost of serving the load, cost of subsidies and environmental costs.)

Answer: 257 + 0.2*257 + 100 = $408.

a.2) Suppose that in order to accomplish his goals of having a system with 6 solar panels, President Sanders needs to provide the same amount of subsidies that Dear Leader Trump needs to provide the coal industry. However, scientists estimate that the environmental costs of this cleaner system are $50 per day. Calculate the total societal cost of running this system (Hint: the sum of cost of serving the load, cost of subsidies, fixed cost of solar panels, and environmental costs.)

<u>Answer: 132 + 0.2*257 + 50 +60 = $293.</u>